Oliver Gross | Research Statement

My research identifies the geometric and topological structures underlying physical and engineered systems and turns them into computational models and algorithms, with applications spanning *fluid* and *plasma dynamics*, *elasticity* and *soft matter*, *locomotion*, and *robotics*. Advances in simulation and scientific machine learning have made computation an engine of discovery across science and engineering. For increasingly complex physical systems, however, adding variables and relying on brute-force computation can demand prohibitive resources, rendering the underlying problem computationally intractable. Geometry provides a complementary path by exposing the mathematical structure that makes this complexity manageable and guides the design of efficient algorithms. In short,

I use geometry to make hard simulation problems tractable.

Grids, meshes, and learned latent variables represent states the physics forbids just as readily as those it permits. As a result, conventional algorithms often compute in a space much larger than the set of physically possible states, then expend substantial effort enforcing the laws that bring the solution back into that set. This can introduce unnecessary variables, numerical stiffness, and even unphysical solutions. This is particularly important in high-stakes applications, such as aerospace navigation, safety-critical robotic autonomy, or fusion-plasma confinement, and poses a major challenge in modern computing: how can algorithms efficiently navigate constrained configuration spaces while respecting their defining geometry, topology, symmetries, and conservation laws?

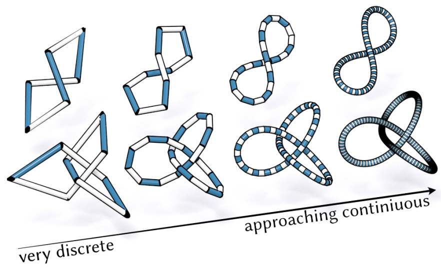

Fig. 1. *My recent work defines discrete elastica without requiring auxiliary discretizations [P5].*

As a computational mathematician working across *differential geometry*, *geometric mechanics*, the *calculus of variations*, *discrete differential geometry (DDG)*, and *physical simulation*, *I take a unified, first-principles approach to continuous formulation, discretization, and numerical solution*. Integrating these stages reveals hidden structure obscured when they are considered separately and distinguishes genuine physical complexity from artifacts of representation. From this structural understanding, *I derive simpler formulations, natural generalizations, and new algorithms that bring otherwise intractable problems within computational reach*. My work has produced advances in differential geometry (Fig. 1) and computational methods for fluids, plasmas, and locomotion [P1, P2, P4, P6, P12]. *Mathematical visualization is also a distinctive part of my research and scientific communication*: I use it to make new geometric structures and ideas transparent. This work has led to an invited review on mathematical films, five years as co-editor of the "Discretization in Geometry and Dynamics" Calendar, and a differential geometry textbook [P11] that ranks #7 by downloads among Springer's more than 7,500 graduate mathematics titles.

My research is organized around three specific themes:

- **A** I develop variational theories of **constrained equilibria and self-organization**, determining how topology, conserved momenta, material constraints, and effective geometry select admissible states.
- **B** I develop the intrinsic geometry and geodesic theory of admissible configuration spaces to formulate **navigation, optimal-control, and inverse-design problems** directly on them.
- **C** I develop **continuous and discrete variational theories** together, determining which continuum structures admit exact discrete counterparts and how discrete models reveal new smooth formulations and correspondences.

Together, these themes build physical structure into computation from the outset, a goal shared by geometric integration [24], compatible discretization [1], nonlinear model reduction [11], and scientific machine learning [22]. Over the next five years, I will pursue this agenda through three connected thrusts.

1 Topology-Aware Methods for Plasma Physics

2 Geometric Methods for Inverse Design in Locomotion

3 Variational Geometry Across Theory and Discretization

Their shared focus on geometry lets methods and insights circulate across the group, while each thrust supports an independent doctoral project. Together, the thrusts form a coherent program for major investigator-led funding, with individual projects suited to more specialized grants. Collaborations I am developing in computer graphics, robotics, and plasma physics provide credible routes to interdisciplinary funding while extending the program's geometric methods into those fields.

1 | Topology-Aware Methods for Plasma Physics

Magnetohydrodynamics (MHD) provides a foundational model for plasma dynamics in astrophysics and magnetic-confinement fusion. A large class of mathematical problems concerns the characterization of field-line configurations of the magnetic field. Like many natural equilibria, they arise as minimizers of an energy, in this case the magnetic energy, under the additional constraint of fixed field-line topology. The resulting *force-free fields* (Fig. 2) are divergence-free fields B satisfying the nonlinear equation $(\operatorname{curl} B) \times B = 0$. Open questions concerning these fields range from the existence and regularity of constrained equilibria to current-sheet formation [28], topology change, and helicity transfer through reconnection [21, 30]. **My research addresses these questions through topology-aware discretization (C)**, seeking transfer laws that persist under refinement and inform a continuous theory.

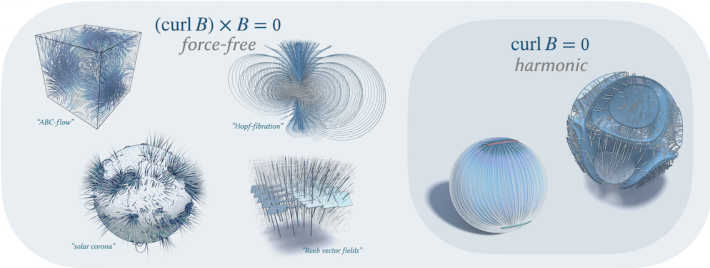

Fig. 2. *Force-free fields emerge across fluid dynamics, plasma physics, and geometry. Their conformal geodesibility connects these settings, identifying field lines with geodesics of a rescaled metric [P2].*

Past accomplishments | I have advanced both the continuous theory and topology-preserving computation of magnetic relaxation [P2, P6, P10]. In three dimensions, such fields are *geodesible*, meaning their field lines are geodesics of some metric [9, 31, 37]. **Although closely related correspondences fail in higher dimensions** [6], **I proved that a stronger version of this correspondence holds in three and higher dimensions** [P2]:

Theorem: Force-free fields are conformally geodesic and vice versa.

A field is *conformally geodesic* if its field lines are geodesics for a conformally related metric. This viewpoint unifies magnetic-energy minimization for stationary plasma fields with mass-norm minimization in geometric measure theory, where geodesics and minimal submanifolds are the stationary objects.

This conformal viewpoint first emerged in my computational treatment of a further abstraction of magnetic relaxation, in which the field is confined to tubular neighborhoods of knotted and linked curves [26]. Relaxation then becomes energy descent on the space of interacting tubes with fixed topology and finite thickness (Fig. 3) **(B)**. Earlier methods based on constrained “knot tightening” represented thickness by fixed radii and interactions through explicit constraints [2, 32]. I resolved these decade-old limitations by encoding tube geometry and interactions entirely in a time-dependent conformal metric. **The resulting geometric flow was the first to handle collision, contact, and spatially and temporally varying thickness without explicit constraints** [P6, P10]. It advects magnetic flux while preserving field-line connectivity, yielding a structure-preserving discretization of a frozen-in field **(A)**.

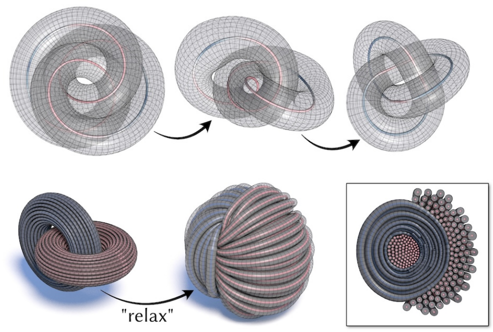

Fig. 3. *Topology-preserving magnetic-energy relaxation of a plasma knot* (top) *and a Hopf link discretized by 100 filaments per component* (bottom) *[P6].*

Future program | A central open problem is a geometric theory predicting when and how magnetic reconnection changes field-line topology. Reconnection locally reorganizes connectivity, interconverting linking, writhe, and twist while redistributing helicity and releasing magnetic energy. Flux-tube theory and analogue experiments resolve special cases [23, 34], while structure-preserving finite-element computations show that local helicity conservation imposes stronger topological barriers than global conservation [10, 19]. Yet how these constraints govern topology change during magnetic relaxation remains unresolved. **My long-term goal is a variational theory characterizing admissible topology change and geometric transfer in magnetic reconnection (A).** This theory will reveal how field-line topology changes and how helicity and magnetic energy are redistributed in nonlinear MHD. As concrete steps, I will extend the geometric theory of plasma knots to nontrivial helicity (Fig. 4) and develop a GPU-parallel solver for resolution-controlled energy spectra, from isolated knots and links to dense filament systems approximating smooth fields **(C)**, enabling fast, reliable, and scalable computation for complex filament systems. This framework will track how reconnection redistributes helicity among linking, writhe, and twist across scales. As in the earlier passage from discrete curve shortening to the smooth conformal correspondence, refinement-stable transfer laws will guide a continuous theory of reconnection in stellar and fusion plasmas.

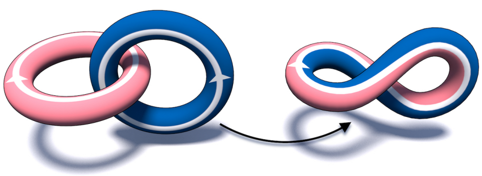

Fig. 4. *Reconnection converts mutual linking into flux-tube twist.*

2 | Geometric Methods for Inverse Design in Locomotion

Locomotion in shape-changing animals and robots couples deformation, rigid pose, and environmental response in a high-dimensional configuration space, making its prediction and design a central problem in robotics and motion planning. Existing approaches fall into two camps. Spacetime optimization and reinforcement learning are currently the only approaches that scale, but offer no intrinsic guarantees of geometric or physical consistency [20, 25, 40]. Geometric mechanics provides such guarantees by construction (Fig. 5) [15, 33, 35], but scales poorly [41]. The central challenge is to develop flexible models that combine the scalability of optimization and learning with the guarantees of geometric formulations across diverse systems, dynamics, and environments **(B)**.

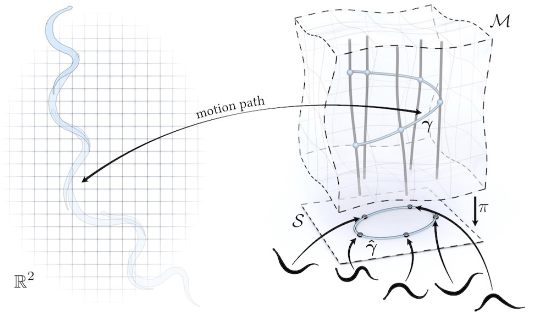

Fig. 5. *In geometric models of locomotion, gauge connections reconstruct a snake's motion from its path through shape space [35].*

Past accomplishments | My contributions carry geometric locomotion from environmental modeling [P3] through motion reconstruction [P8, P13] to intrinsic gait design [P1, P4]. **I developed a unified framework for motion reconstruction across inertial, dissipative, and externally forced regimes (C)**. It reconstructs world trajectories from prescribed shape sequences [P8] and incorporates moving media and external forces through local lift, drag, and added-mass laws [P13]. Admissible velocities carry an intrinsic metric encoding internal deformation and environmental dissipation **(B)**.

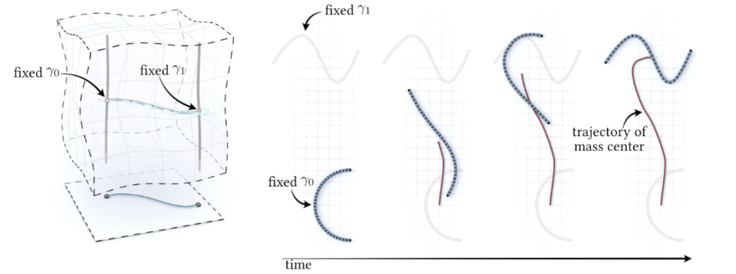

Fig. 6. *Sub-Riemannian formulations make optimal and energy-efficient gaits computable as geodesics tangent to the admissible velocity distribution.*

The corresponding inverse problem asks which shape changes realize a target motion and which do so most efficiently. I cast each as a geodesic problem on a sub-Riemannian manifold, whose metric is defined only on a distribution of admissible directions, and developed general differentiable solvers for optimal locomotion and inverse gait design. Least-dissipation gaits are sub-Riemannian geodesics subject to fixed endpoints, cyclic shape change, or prescribed net rigid motion [27, P4] **(C)**. **The resulting geodesics match experimental observations strikingly well**. They recover characteristic snake and spermatozoon motions and reveal membrane-deformation gaits that outperform established kinematic optima. Moreover, the inverse-design solvers outperform state-of-the-art geometric methods in terms of solver efficiency and scalability, while capturing more phenomena [41, P1].

Once gait design is tractable, the next problem is to quantify how model-reduction and sim-to-real errors perturb reconstructed motion. In recent work, I used geometric models to localize these two error sources in a snake-like robot [P3]. With collaborators at MIT and UTokyo, **I am extending the framework from prescribed to compliant shape change (B, C)**. Again, local interaction laws couple actuation, deformation, and fluid forces without volumetric simulation, while differentiability enables joint optimization of gait, morphology, and material properties.

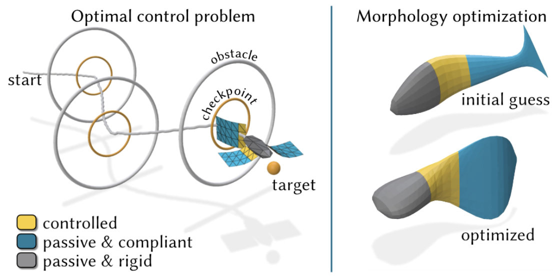

Fig. 7. *Preliminary results from my work on geometric inverse design for soft-body locomotion.*

Future program | A central mathematical problem is to characterize the intrinsic structures governing nonholonomic locomotion with inertia, dissipation, forcing, or compliance and determine what survives model reduction and discretization. **I will organize these structures into compatible geometric hierarchies for prediction, optimal control, and inverse design across model fidelities (B, C)**. Using geometric mechanics, I will derive corresponding reconstruction laws and variational formulations, clarify the passage between Riemannian and sub-Riemannian descriptions, and establish existence and regularity results for the resulting minimizers. Prolongation and restriction maps will connect resistive-force, finite-element, material-point, and reduced-order models while preserving admissible directions and commuting with pose reconstruction. I will use these maps to establish convergence and derive error estimates for structure-preserving discretizations while relating reachable sets, geodesics, and optimal costs across levels. These guarantees will let reinforcement learning search inexpensive models while reserving high-fidelity simulation for promising policies. With Michael Tolley at UC San Diego, I will test and refine this theory on a modular eel-inspired soft robot [7].

3 | Variational Geometry Across Theory and Discretization

Variational principles characterize fundamental curves and surfaces in differential geometry, including Euler elastica, minimal surfaces, and Willmore surfaces. These objects carry rich mathematical structure and admit equivalent characterizations that are often lost in specific discretizations or computational schemes. This raises a natural question: How do we model these differential-geometric objects so that most of their mathematical structures persist under discretization? A parallel question asks which of these structures persist under the weakest possible regularity assumptions **(C)**.

Past accomplishments | My contributions span geometric invariance in curve families [P9], application-driven extensions of smooth curve theory [P7], discrete variational models for curves [P5], and variational geometry for differential forms [P2]. Elastica, the equilibrium shapes of slender rods, connect variational geometry to pendulum dynamics, vortex-filament flow, and the nonlinear Schrödinger equation. Applications demand richer models that accommodate variable stiffness, curved rest shapes, and external loading [3, 12, 13], raising a fundamental question **(A, C)**: which structures of classical theories survive these added complexities? For variable bending stiffness (Fig. 8), **I showed that these connections persist as variable-length pendulum dynamics and finite-thickness vortex-filament flow** [P7].

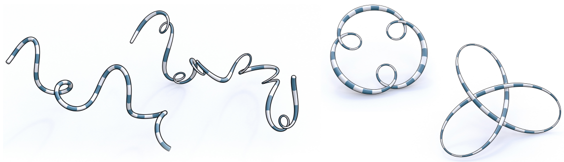

Fig. 8. *Examples of elastic curves with variable bending stiffness [P7].*

Discrete elastica pose a basic challenge because polygonal curves lack canonical curvature. Existing constructions address this ambiguity either by discretizing curvature and material frames [4], or by discretizing associated integrable dynamics [5, 17, 18]. Building on a largely undeveloped isoperimetric characterization of elastica as stationary points of length at fixed area and volume vectors [8], **I developed a systematic geometric-mechanics theory by identifying these vectors as rigid-motion momenta (A)** and restricting the formulation canonically to polygons, **yielding a lower-order, curvature-free definition of discrete elastica (C).** Using first-order integrals lowers regularity requirements, admits piecewise-linear elements, and improves reduced-Hessian condition-number scaling from N^4 to N^2, while retaining quadratic convergence and coarse-resolution fidelity (Fig. 1) [P5] **(B)**. The construction also carries the Marsden–Weinstein form and its Hamiltonian flows to polygonal curves. Both projects on elastic curves originated in student theses I co-supervised.

The conformal viewpoint of Thrust 1 extends from geodesic field lines to the variational geometry of foliations by minimal submanifolds. A simple, closed, nowhere-vanishing k-form defines an $(n-k)$-dimensional foliation. As in [P2], **an explicit conformal change of metric makes the defining form stationary for energy exactly when its foliation is stationary for mass, under the same admissible variations (A, C)**. Under homological or closedness constraints, the critical fields are harmonic and eikonal, connecting these energy and mass formulations to Beckmann optimal transport and calibrated geometry. Sullivan characterized which foliations admit metrics making their leaves minimal [38], while Harvey–Lawson calibrations certify minimality under stronger conditions and support differential-form computations of minimal surfaces [14, 39]. Because stationarity need not imply calibration, I am developing a generalized calibration admitting transverse twist, mirroring the passage from harmonic to general force-free fields **(C)**. I am also developing a complementary momentum formulation, with preliminary results identifying isotopy-constrained criticality with a vanishing momentum condition.

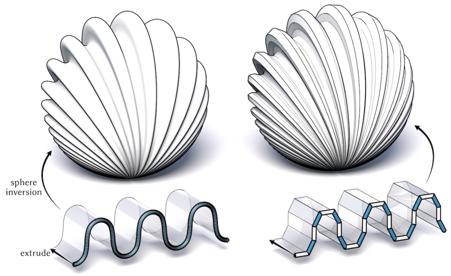

Fig. 9. *Willmore surfaces obtained from analogous continuous* (left) *and semi-discrete* (right) *constructions.*

Future program | Across curves, surfaces, and differential forms, I will use geometric structure to guide discretization and discrete behavior to sharpen smooth theory. **My long-term goal is to organize this exchange into compatible variational hierarchies across ambient geometries, dimensions, and discretizations (C)**. By converting symmetry, conservation, and recurrence into lower-order formulations and better-conditioned solvers, these hierarchies will yield intrinsic methods for otherwise intractable problems. For curves, I will replace Euclidean momenta by those generated by spherical and hyperbolic isometries, extend planar space-form theory to spatial elastica, and analyze their commuting flows, stability, and bifurcations [8, 18]. For surfaces, my coauthored textbook [P11] develops the variational arc from elastica and their Willmore cylinders to general Willmore surfaces, while Pinkall's Hopf construction and its constrained extensions lift spherical elastica to constrained-Willmore tori in S^3 [16, 29]. I will lift discrete spherical elastica to semidiscrete constrained-Willmore tori and determine which variational and integrable structures persist beyond the Hopf ansatz on general triangulated surfaces [36]. For differential forms, I will determine how transverse twist modifies minimality and calibration, deepen the connection between force-free fields and minimal-submanifold geometry, and identify corresponding structures in optimal transport. I will then discretize these structures and develop computational tools for the resulting fields and foliations.

Publications

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[P3] S. Even, P. S. Martinez, C. Keogh, O. Gross, Y. Ozkan-Aydin, and P. Schröder, "A Variational Approach to Geometric Mechanics for Undulating Robotic Locomotion," 2024, arXiv: 2409.10827.

[P4] O. Gross, F. Hartwig, M. Rumpf, and P. Schröder, "Sub-Riemannian Boundary Value Problems for Optimal Geometric Locomotion," 2026, arXiv: 2602.12199.

[P5] O. Gross, R. Jammula, and A. Chern, "Elastic Curves via Geometric Mechanics," 2026, arXiv: 2607.29654.

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