Oliver Gross | Research Statement

My research in *geometric computing* develops *structure-preserving algorithms for physical simulation and inverse design*. I combine differential geometry, geometric mechanics, and scientific computing to study how geometry and physical constraints govern equilibria, motion, and form in physical and engineered systems, and to develop mathematical models and algorithms from the resulting structure. Its applications span *fluid and plasma dynamics*, *elasticity and soft matter*, *locomotion*, and *robotics*. Advances in simulation and scientific machine learning have made computation an engine of discovery across science and engineering. For increasingly complex systems, however, resolving more physical detail by simply adding variables and computational power quickly becomes prohibitive. Geometry identifies the admissible states, symmetries, conservation laws and variational principles that organize the dynamics, providing the structure needed to construct detailed yet tractable models. In short,

I use geometry to make complex physical problems tractable for theory and computation.

Physical systems generally occupy only a constrained subset of their apparent configuration space. Grids, meshes and learned latent variables, however, represent states that the physics forbids just as readily as those it permits. Algorithms must then recover the missing physical structure through constraints or penalties, introducing unnecessary variables, numerical stiffness or unphysical solutions. This leads to the central question of my program: how can continuous models, discretizations and data-informed approximations be connected across levels of fidelity while preserving the geometry, topology, symmetries and conservation laws that govern the system?

Working across *differential geometry*, *geometric mechanics*, the *calculus of variations*, *discrete differential geometry (DDG)*, and *physical simulation*, *I take a unified, first-principles approach to continuous formulation, discretization, and numerical solution*. Integrating these stages reveals hidden structure obscured when they are considered separately and distinguishes genuine physical complexity from artifacts of representation. From this structural understanding, *I derive simpler formulations, natural generalizations, and new algorithms that bring otherwise intractable problems within computational reach*. My work has produced advances in differential geometry (Fig. 1), mathematical physics, and computational methods for fluids, plasmas, and locomotion [P1,P2,P4,P6,P12]. *Mathematical visualization is also a distinctive part of my research and scientific communication*: I use it to make new geometric structures and ideas transparent. This work has led to an invited article on mathematical films, five years as co-editor of the "Discretization in Geometry and Dynamics" Calendar, and a differential geometry textbook [P11] that ranks **7th by downloads among Springer's more than 7,500 graduate mathematics titles**.

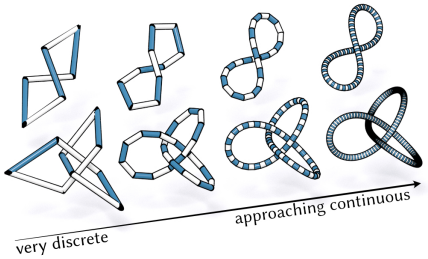


Fig. 1. *My recent work defines discrete elastica without requiring auxiliary discretizations [P5].*

My research is organized around three specific themes:

A I develop variational theories of **constrained equilibria and self-organization**, determining how topology, conserved momenta, material constraints, and effective geometry select admissible states.

B I develop the intrinsic geometry and geodesic theory of admissible configuration spaces to formulate **navigation, optimal-control, and inverse-design problems** directly on them.

C I develop **continuous and discrete variational theories** together, determining which continuum structures admit exact discrete counterparts and how discrete models reveal new smooth formulations and correspondences.

Together, these themes build physical structure into computation from the outset, a goal shared by geometric integration [24], compatible discretization [1], nonlinear model reduction [11], and scientific machine learning [22]. Over the next five years, I will pursue this agenda through three connected thrusts.

1 **Topology-Aware Methods for Plasma Physics**

2 **Geometric Methods for Inverse Design in Locomotion**

3 **Variational Principles Across Theory and Discretization**

Their shared focus on geometry lets methods and insights circulate across the group, while each thrust supports an independent doctoral project. The program is structured for an NSF CAREER proposal (NSF 22-586). I would pursue focused projects in structure-preserving numerical methods through the DMS Computational Mathematics program (NSF 26-520) and in geometric modeling and simulation for computer graphics through NSF CISE's Human-Centered Computing program (NSF 25-543). Its foundational questions align with the Simons Collaborations in Mathematics and the Physical Sciences, while I am actively participating in collaborations in scientific computing, computer graphics, robotics, and plasma physics that create pathways to the more specialized ARO and AFOSR programs discussed below.

1 | Topology-Aware Methods for Plasma Physics

Magnetohydrodynamics (MHD) provides a foundational model for plasma dynamics in astrophysics and magnetic-confinement fusion. A large class of mathematical problems concerns magnetic field-line configurations. Like many natural equilibria, they arise as minimizers of an energy, in this case the magnetic energy, under the additional constraint of fixed field-line topology **(A)**. The resulting *force-free fields* (Fig. 2) are divergence-free fields B satisfying the nonlinear equation $(\operatorname{curl} B) \times B = 0$. Open questions range from the existence and regularity of constrained equilibria to current-sheet formation [28], topology change, and helicity transfer through reconnection [21, 30]. **My research addresses these questions through topology-aware discretization (C)**, seeking transfer laws that persist under refinement and inform a continuous theory.

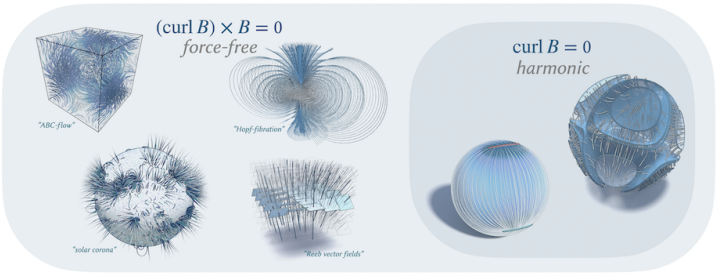


Fig. 2. *Force-free fields emerge across fluid dynamics, plasma physics, and geometry. Their conformal geodesibility connects these settings, identifying field lines with geodesics of a rescaled metric [P2].*

Past accomplishments | I have advanced both the continuous theory and topology-preserving computation of magnetic relaxation [P2, P6, P10]. In three dimensions, such fields are *geodesible*, meaning their field lines are geodesics of some metric [9, 31, 37]. **Overcoming limitations of earlier results** [6], **I proved that a stronger version of this correspondence holds in three and higher dimensions** [P2]:

Theorem: Force-free fields are conformally geodesic and vice versa.

A field is *conformally geodesic* if its field lines are geodesics for a conformally related metric. This viewpoint unifies magnetic-energy minimization for stationary plasma fields with mass-norm minimization in geometric measure theory, where geodesics and minimal submanifolds are the stationary objects.

It first emerged in my computational treatment of a further abstraction of magnetic relaxation, in which the field is confined to tubular neighborhoods of knotted and linked curves [26, P6]. Relaxation then becomes energy descent on the space of interacting tubes with fixed topology and finite thickness. Earlier methods based on constrained "knot tightening" relied on fixed tube thickness and interactions through explicit constraints [2, 32]. I resolved these decade-old limitations by encoding tube geometry and interactions entirely in a time-dependent conformal metric. **The resulting geometric flow was the first to handle collision, contact, and spatially and temporally varying thickness without explicit constraints** [P6, P10]. It advects magnetic flux while preserving field-line connectivity, yielding a structure-preserving discretization of a frozen-in field **(C)** (Fig. 3).

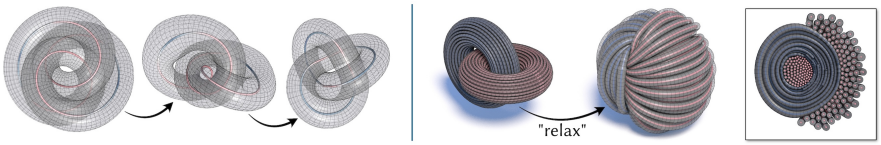


Fig. 3. *Topology-preserving magnetic-energy relaxation of a plasma knot* (left) *and a Hopf link discretized by 100 filaments per component* (right) *[P6].*

Future program | A central open problem is a geometric theory predicting when and how magnetic reconnection changes field-line topology. Reconnection locally reorganizes connectivity, interconverting linking, writhe, and twist while redistributing helicity and releasing magnetic energy. Flux-tube theory and analogue experiments resolve special cases [23, 34], while structure-preserving finite-element computations show that local helicity conservation imposes stronger topological barriers than global conservation [10, 19]. Yet how these constraints govern topology change during magnetic relaxation remains unresolved. **My long-term goal is a variational theory characterizing admissible topology change and geometric transfer laws for reconnection (A)**. Such a theory directly addresses ARO's Complex Dynamics and Systems program (ARL-BAA-0018). As a first project, I will extend the geometric theory of plasma knots to nontrivial helicity (Fig. 4), then develop a GPU-parallel solver for resolution-controlled energy spectra, progressing from isolated knots and links to dense filament systems approximating smooth fields. The solver advances the emphasis of AFOSR's Computational Mathematics program (FA955026S0001) on fast, reliable, and scalable algorithms for complex physical systems. I will subsequently study how reconnection interconverts linking, writhe, and twist, seeking transfer laws that persist under refinement and can guide new continuous theories of reconnection in stellar and fusion plasmas **(C)**.

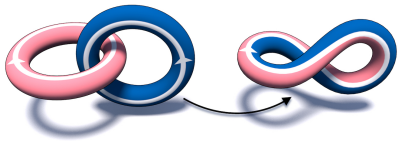


Fig. 4. *Reconnection converts mutual linking into flux-tube twist.*

2 | Geometric Methods for Inverse Design in Locomotion

Locomotion in shape-changing animals and robots couples deformation, rigid pose, and environmental response in a high-dimensional configuration space, making its prediction and design a central problem in robotics and motion planning. Existing approaches fall into two camps. Spacetime optimization and reinforcement learning are currently widely used scalable approaches, but offer no intrinsic guarantees of geometric or physical consistency [20,25,40]. Geometric mechanics provides such guarantees by construction (Fig. 5) [15,33,35], but scales poorly [41]. The central challenge is to develop flexible models that combine the scalability of optimization and learning with the guarantees of geometric formulations across diverse systems, dynamics, and environments **(B)**.

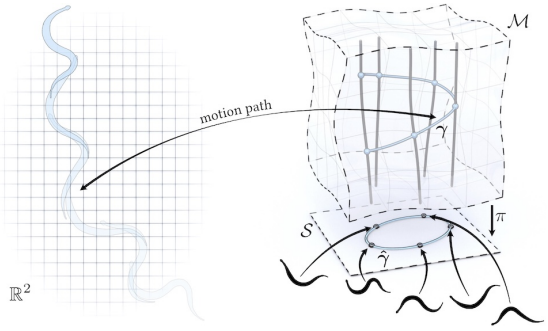


Fig. 5. *In geometric models of locomotion, gauge connections reconstruct a snake's motion from its path through shape space [35].*

Past accomplishments | My contributions carry geometric locomotion from environmental modeling [P3] through motion reconstruction [P8,P13] to intrinsic gait design [P1,P4]. **I developed a unified framework for motion reconstruction across inertial, dissipative, and externally forced regimes (C)**. It reconstructs world trajectories from prescribed shape sequences [P8] and incorporates moving media and external forces through local lift, drag, and added-mass laws [P13]. Admissible velocities carry an intrinsic metric encoding internal deformation and environmental dissipation **(B)**.

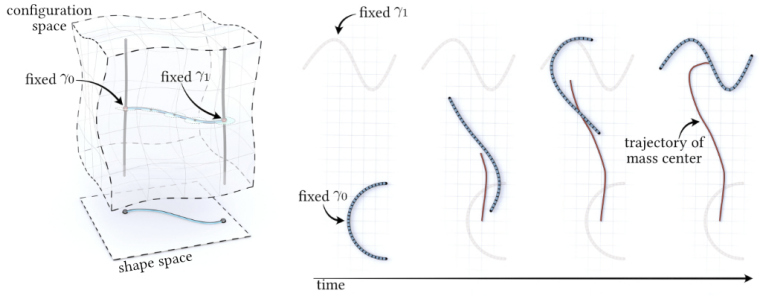


Fig. 6. *Sub-Riemannian formulations make optimal and energy-efficient gaits computable as geodesics tangent to the admissible velocity distribution.*

The corresponding inverse problem asks which shape changes realize a target motion and which do so most efficiently. I cast each as a geodesic problem on a sub-Riemannian manifold, whose metric is defined only on a distribution of admissible directions, and developed general differentiable solvers for optimal locomotion and inverse gait design. This shifts computation from curvature-based descriptions of displacement to entire admissible trajectories, with physical constraints built into their geometry. Least-dissipation gaits are sub-Riemannian geodesics subject to fixed endpoints, cyclic shape change, or prescribed net rigid motion [27,P4] **(C)**. **The resulting geodesics match experimental observations strikingly well**. They recover characteristic snake and spermatozoon motions and reveal membrane-deformation gaits that outperform established kinematic optima, while accommodating more than two orders of magnitude more shape degrees of freedom than recent geometric methods specifically targeting high-dimensional motion planning [41,P1].

Once gait design is tractable, the next problem is to quantify how model-reduction and sim-to-real errors perturb reconstructed motion. In recent work, I used geometric models to localize these two error sources in a snake-like robot [P3]. With collaborators at MIT and UTokyo, **I am extending the framework from fully prescribed to underactuated shape changes (B, C)**. Again, local interaction laws couple actuation, deformation, and fluid forces without volumetric simulation, while differentiability enables joint optimization of gait, morphology, and material properties.

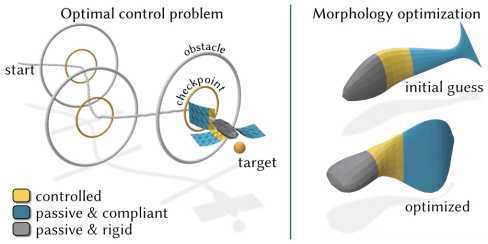


Fig. 7. *Preliminary results from my work on geometric inverse design for soft-body locomotion.*

Future program | A central mathematical problem is to determine when models of the same physical system at different levels of fidelity *reproduce the same admissible motions and optimal gaits*. Here, fidelity can be reduced along two complementary axes: the representation of shape, from fully resolved deformable bodies through reduced-order models to skeletal descriptions, and the physical model, from continuum discretizations such as FEM and MPM to simplified models of body–fluid interaction. I will construct prolongation and restriction maps across both hierarchies and determine when they preserve the geometric structures governing locomotion. Compatibility with momentum maps will guide these constructions: I will determine which symmetries, admissible motions, and dissipation mechanisms survive each approximation and derive error estimates where possible **(A, B)**. This framework will let inexpensive models screen gaits, morphologies, and control policies, including those obtained through reinforcement learning, before high-fidelity simulation and experimental testing. Experiments with robots such as modular eel-inspired platforms [7] will test how reliably these geometric models predict and optimize physical locomotion.

3 | Variational Principles Across Theory and Discretization

Variational principles characterize fundamental curves and surfaces in differential geometry, including Euler elastica, minimal surfaces, and Willmore surfaces. These objects carry rich mathematical structure and admit equivalent characterizations that are often lost in specific discretizations or computational schemes. This raises a natural question: How do we model these differential-geometric objects so that most of their mathematical structures persist under discretization? A parallel question asks which of these structures persist under the weakest possible regularity assumptions **(C)**.



Fig. 8. *Examples of elastic curves with variable bending stiffness [P7].*

Past accomplishments | My contributions span geometric invariance in curve families [P9], application-driven extensions of smooth curve theory [P7], discrete variational models for curves [P5], and variational geometry for differential forms [P2]. Elastica, the equilibrium shapes of slender rods, connect variational geometry to pendulum dynamics, vortex-filament flow, and the nonlinear Schrödinger equation. Applications demand richer models that accommodate variable stiffness, curved rest shapes, and external loading [3, 12, 13], raising a fundamental question **(A, C)**: which structures of classical theories survive these added complexities? For variable bending stiffness (Fig. 8), **I showed that these connections persist as variable-length pendulum dynamics and finite-thickness vortex-filament flow** [P7].

Discrete elastica pose a basic challenge because polygonal curves lack canonical curvature. Existing constructions address this ambiguity either by discretizing curvature and material frames [4], or by discretizing associated integrable dynamics [5,17,18]. Building on a largely undeveloped isoperimetric characterization of elastica as stationary points of length at fixed area and volume vectors [8], **I developed a systematic geometric-mechanics theory by identifying these vectors as rigid-motion momenta (A)** and restricting the formulation canonically to polygons, **yielding a lower-order, curvature-free definition of discrete elastica (C).** Using first-order integrals lowers regularity requirements, admits piecewise-linear elements, and improves the scaling of the condition number from N^4 to N^2, while retaining quadratic convergence and coarse-resolution fidelity (Fig. 1) [P5] **(B)**. The construction also carries the Marsden–Weinstein form and its Hamiltonian flows to polygonal curves. Both projects on elastic curves originated in student theses I co-supervised.

The conformal viewpoint of Thrust 1 extends from geodesic field lines to the variational geometry of foliations by minimal submanifolds. A simple, closed, nowhere-vanishing k-form defines an $(n-k)$-dimensional foliation. As in [P2], **an explicit conformal change of metric makes the defining form stationary for energy exactly when its foliation is stationary for mass, under the same admissible variations (A, C)**. Under homological or closedness constraints, the critical fields are harmonic and eikonal, connecting these energy and mass formulations to Beckmann optimal transport and calibrated geometry. Sullivan characterized which foliations admit metrics making their leaves minimal [38], while calibrations certify minimality under stronger conditions [14, 39]. Because stationarity need not imply calibration, I am developing a generalized calibration theory admitting transverse twist, mirroring the passage from harmonic to general force-free fields **(C)**, together with a complementary momentum formulation where preliminary results identify isotopy-constrained criticality with a vanishing momentum condition.

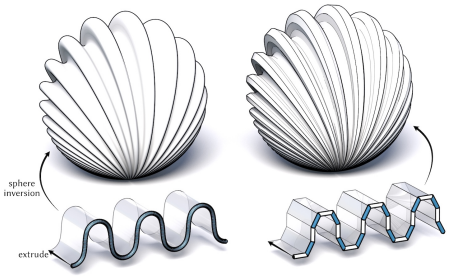


Fig. 9. *Willmore surfaces obtained from analogous continuous* (left) *and semi-discrete* (right) *constructions.*

Future program | Across curves, surfaces, and differential forms, I will use geometric structure to guide discretization and discrete behavior to sharpen smooth theory. **My long-term goal is to identify variational structures that persist across ambient geometries, dimensions, and discretizations (C).** By converting symmetry, conservation, and recurrence into lower-order formulations and better-conditioned solvers, this thrust addresses ARO's Computational Mathematics program (ARL-BAA-0034). My first priority is a momentum-based formulation of spatial elastica in space forms. By replacing Euclidean momenta with those generated by spherical and hyperbolic isometries, I will extend planar space-form theory to spatial elastica, determine which commuting flows persist, and analyze stability and bifurcations [8, 18]. The reach of curve theory extends to surfaces: Fig. 9 shows parallel continuous and semi-discrete constructions of Willmore surfaces [36], while Pinkall's Hopf construction lifts spherical elastica to constrained-Willmore tori in S^3 [16, 29]. I will develop these connections systematically and determine which variational and integrable structures persist in the passage from curves to surfaces and from smooth to (semi-)discrete geometry. In parallel, I will complete the generalized calibration and momentum formulation for differential forms, determine how transverse twist modifies minimality, and discretize the resulting fields and foliations. Extensions to general triangulated surfaces and connections with optimal transport are longer-term goals.

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