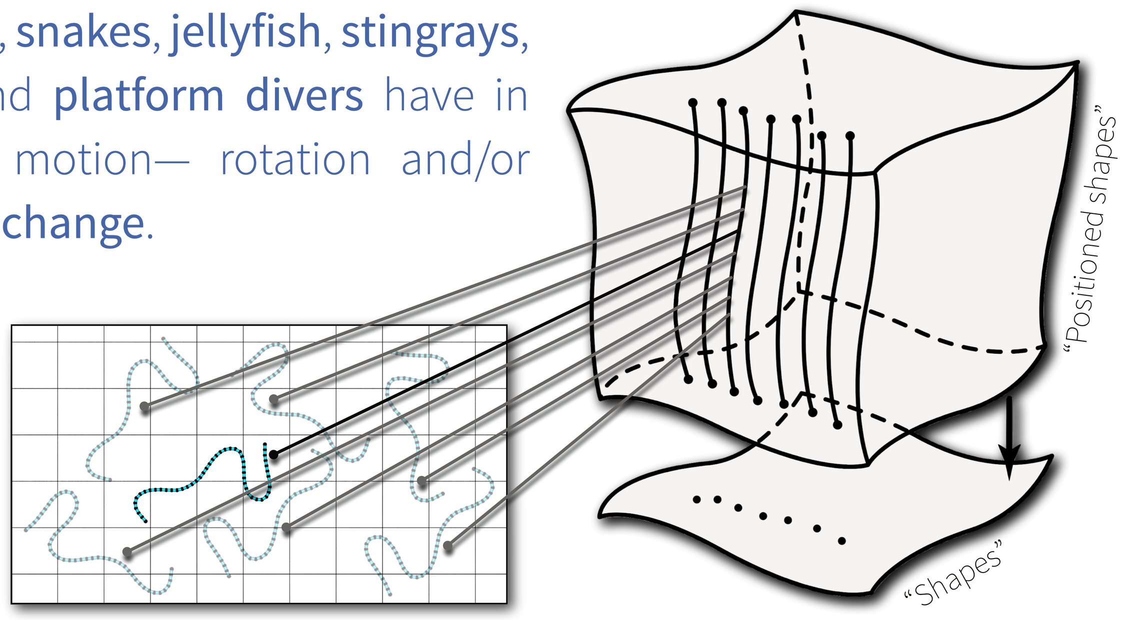
Motion from Shape Change

Oliver Gross[1], Yousuf Soliman[2], Marcel Padilla[1], Felix Knöppel[1], Ulrich Pinkall[1], Peter Schröder[2]
[1]TU Berlin, [2]Caltech

SIGGRAPH 2023
LOS ANGELES+ 6-10 AUG

Motivation and Model

What do **cells**, **spermatozoa**, **snakes**, **jellyfish**, **stingrays**, **falling cats**, **astronauts**, and **platform divers** have in common? They all effect motion— rotation and/or translation—through **shape change**.

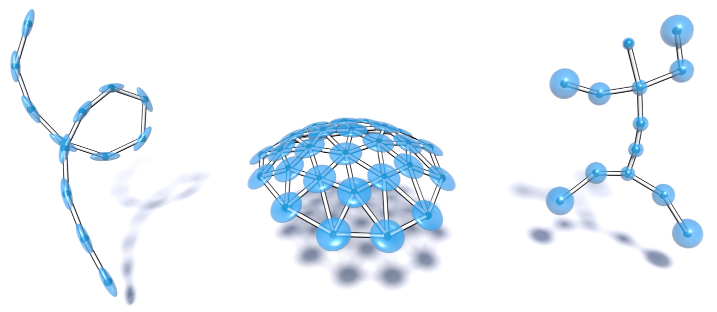


We model motion with the help of the **fiber bundle** of **positioned shapes** (in world space) over the space of **shapes**.

- Each fiber consists of the **Euclidean transformations** $\mathrm{SE}(3)$.
- **Riemannian metric** $\langle\cdot,\cdot\rangle_{\mathcal{B}}$ on $\mathcal{M}$ determines different scenarios

Physical motion $t \mapsto \gamma_t$ is the lift of a 1-parameter family of shapes $t \mapsto S_t$, which is a **stationary point** of

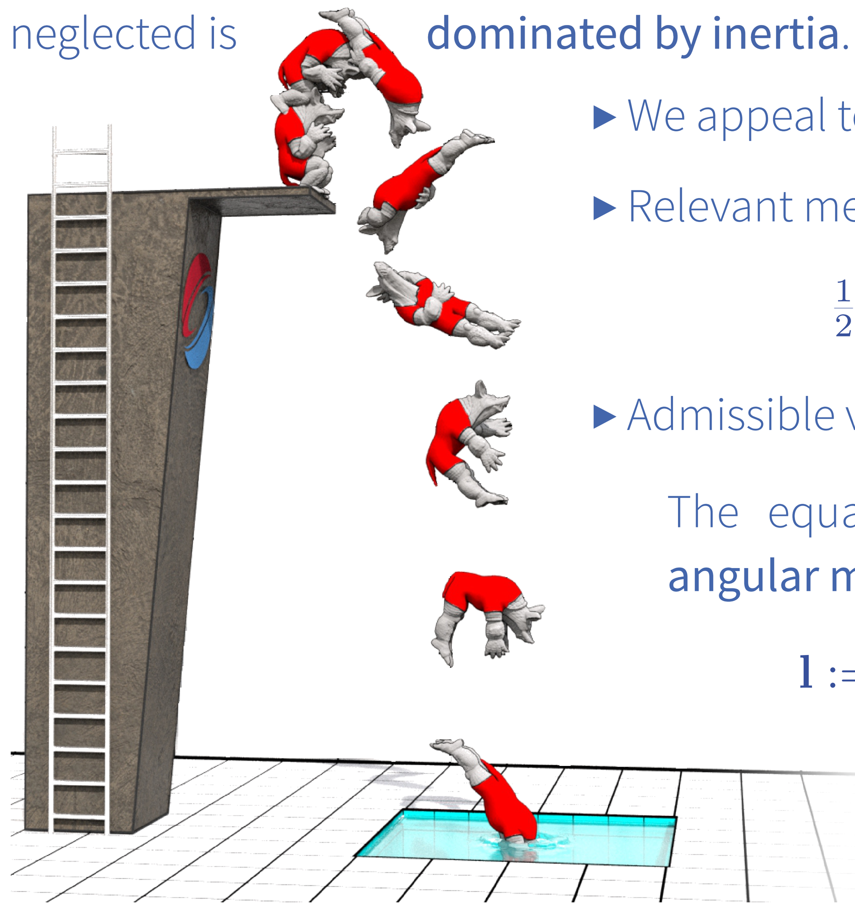
$$\mathcal{E}(\gamma) := \tfrac{1}{2}\int_0^T \langle\gamma',\gamma'\rangle_{\mathcal{B}}\,dt$$



under suitable variations.

Negligible Medium

Motion in a **negligible medium**, such as air or vacuum, whose influence can be neglected is **dominated by inertia**.



- We appeal to **Euler's principle of least action**
- Relevant metric defines **Kinetic energy**

$$\tfrac{1}{2}\int_0^T \langle\gamma',\gamma'\rangle_{\mathcal{K}}\,dt$$

- Admissible variations **fix the endpoints**

The equations of motion state that **linear** and **angular momentum**

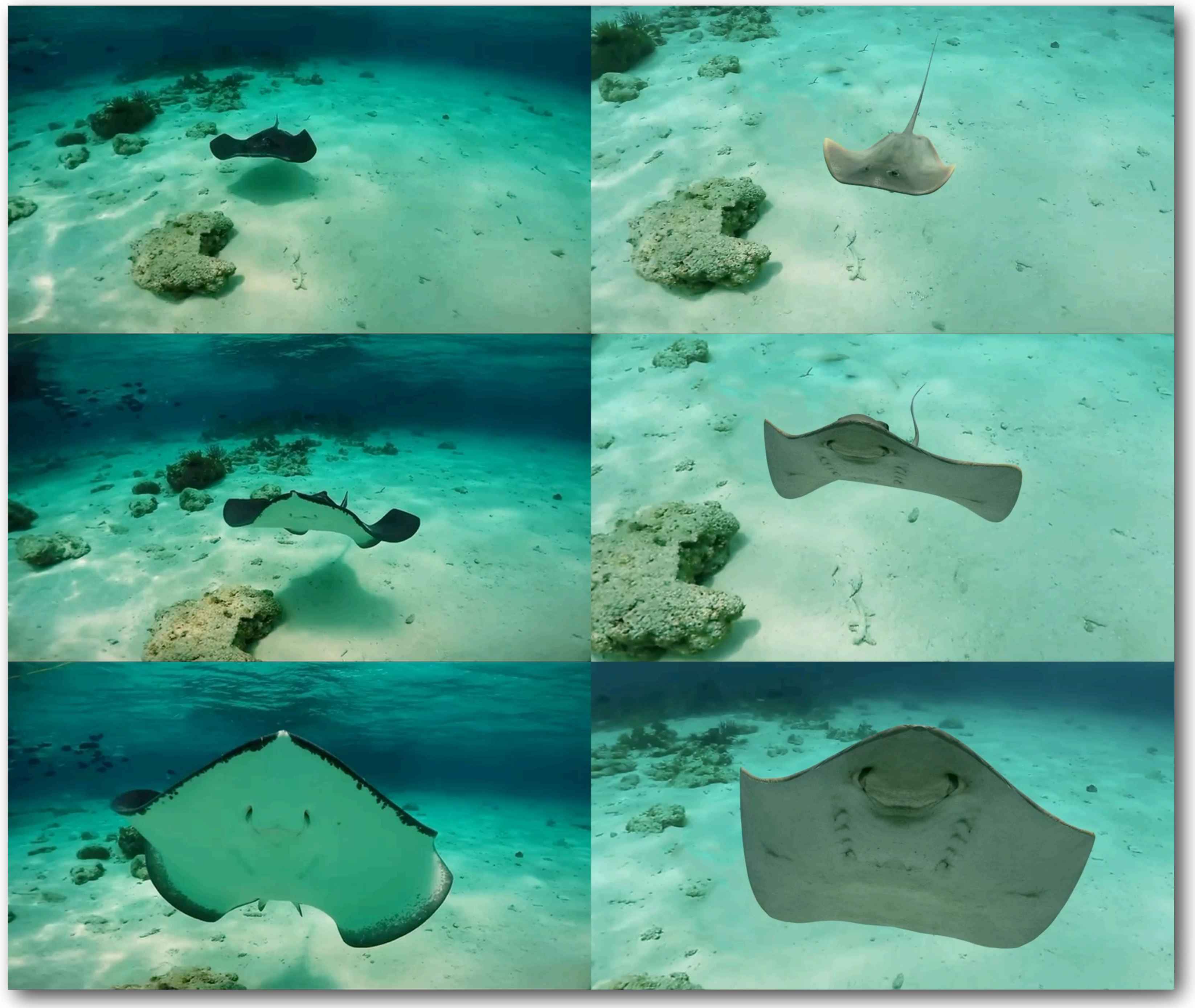
$$\mathbf{l} := \sum_{j=1}^{n} m_j p_j \times p_j' \qquad \mathbf{p} := \sum_{j=1}^{n} m_j p_j'$$

of the shape changing body are constant–but not necessarily zero.

Project Information

The **paper**, an **implementation** of our algorithm in **Houdini** and supplementary **videos** can be found on our project page:

https://olligross.github.io/projects/MotionFromShapeChange/MotionFromShapeChange_project.html

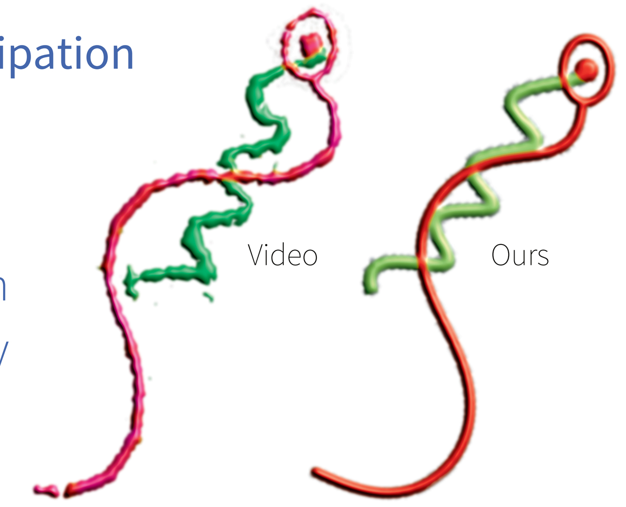


Left: underwater video capture of a sting ray. Right: output of our algorithm based on providing only an undulating surface in the shape of a sting ray.

Highly Viscous Medium

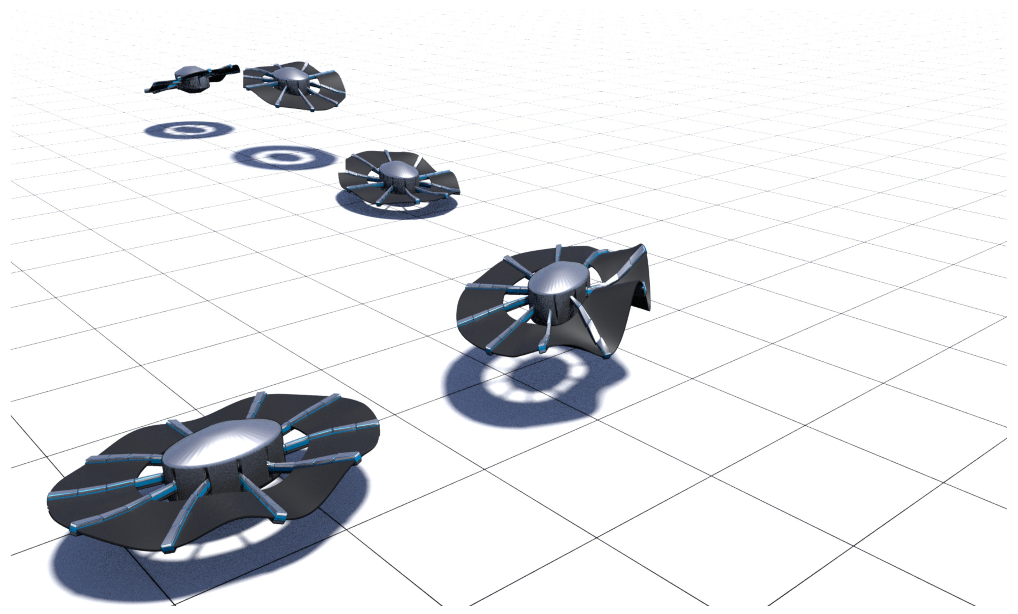
Motion in a **highly viscous** medium, such as for tiny creatures (e.g., bacteria or **spermatozoa**) in water, is dominated by *Rayleigh* ***dissipation***.

- We appeal to **Helmholtz' principle of least dissipation**
- **All** variations are admissible



Theorem: A lift $t \mapsto \gamma_t$ solves the equation of motion for movement in a highly viscous medium if and only if γ_t' is orthogonal to the orbits, i.e.,

$$\gamma_t' \perp T_{\gamma_t} G(\hat{\gamma}_t) \quad \forall t \in [0,T]$$



We use the (local) **dissipation metric**

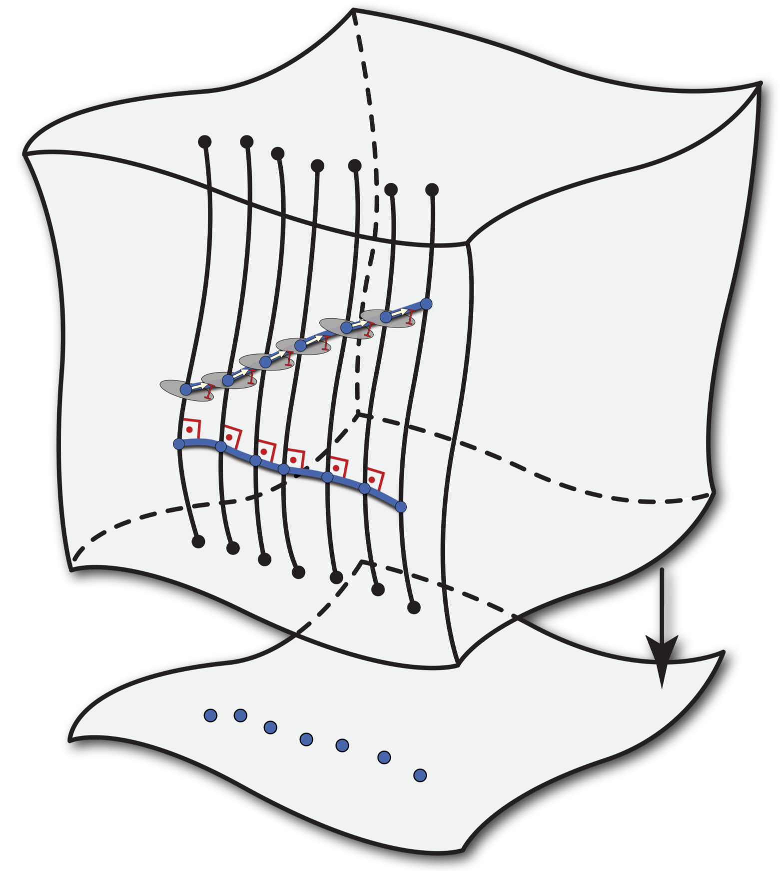
$$D_j := m_j\,(\epsilon I + (1-\epsilon)P_j)$$

controlled by the (local) **anisotropy-parameter** ϵ. It describes the relative ease of tangential motion compared to normal motion.

Algorithm

We take a **time-discrete** sequence $\gamma_0,\ldots,\gamma_T$ of shapes an input to our algorithm. To derive a **variational integrator** we consider stationary points of the **discretized energy**

$$\sum_{k=0}^{T-1}\left\langle \tfrac{1}{2}(B^{\gamma_t}+B^{\gamma_{t-1}})\Delta p^t, \Delta p^t\right\rangle$$



which are charcterized by:

- **zero**-momentum μ (dissipative setup)
- **constant** momentum μ (inertial setup)

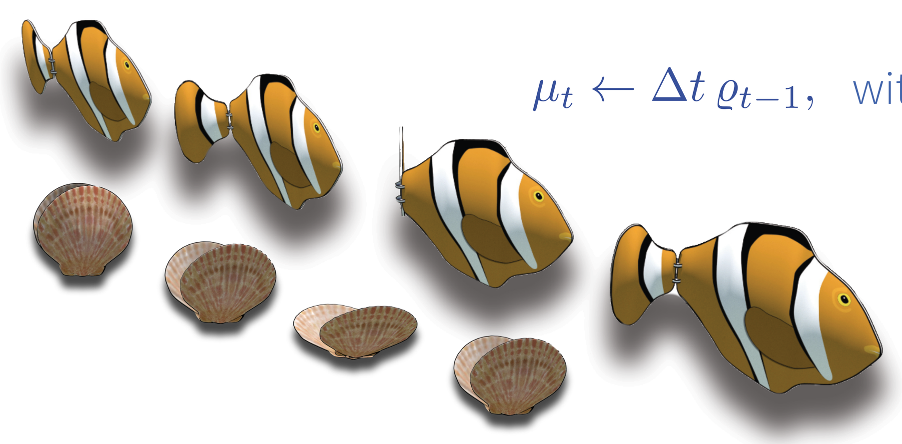
Algorithm 2 — IntegrateMotion$(\mu_0,\hat{\gamma}_0,\ldots,\hat{\gamma}_T)$

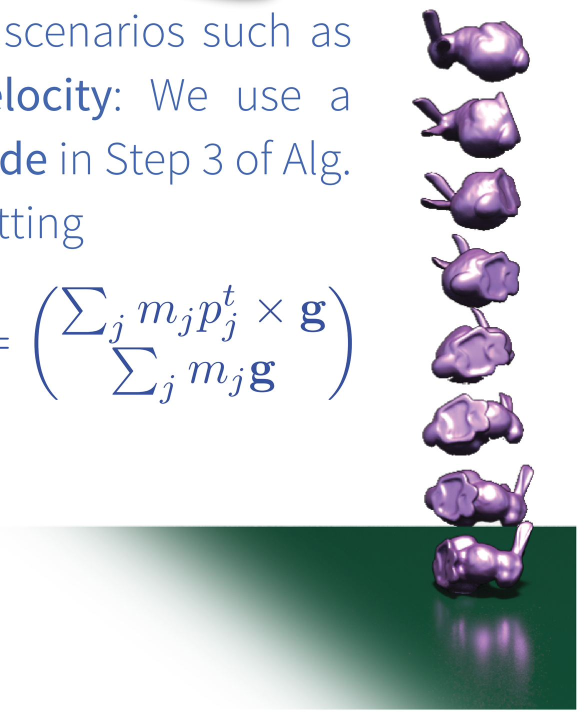
Input: shapes and target momentum $(\mu_0,\hat{\gamma}_0,\ldots,\hat{\gamma}_T)$
Output: positioned shapes $\gamma_0,\ldots,\gamma_T$

```
1: γ_0 ← γ̂_0
2: for t = 1, ..., T do
3:     g_t ← solve DiscreteMomenta(γ_{t−1}, g_t(γ̂_t)) = μ_0
4:     γ_t ← g_t(γ̂_t)
5: end for
```

Our first-order method can be used to accomondate scenarios such as **rigid bodies falling under water** with **terminal velocity**: We use a dissipation metric $\langle\cdot,\cdot\rangle_{\mathcal{D}}$ and **modify the right-hand side** in Step 3 of Alg. 2 to account for the gravitational force and torque by setting

$$\mu_t \leftarrow \Delta t\,\varrho_{t-1}, \quad \text{with} \quad \varrho_t = \begin{pmatrix}\sum_j m_j p_j^t \times \mathbf{g} \\ \sum_j m_j \mathbf{g}\end{pmatrix}$$





Similarly, a **second order** treatment is implemented with **minimal modification** by using the **semi-implicit Euler** scheme:

$$\mu_t \leftarrow \mu_{t-1} + \Delta t\,\varrho_{t-1}$$
$$g_t \leftarrow \text{solve } \mathbf{DiscreteMomenta}\,(\gamma_{t-1}, g_t(\hat{\gamma}_t)) = \mu_t.$$

TU berlin · Caltech · EINSTEIN Foundation.de